

UPSEE 2016
Paper 1 Code AA Solutions
Physics

Ans.1: (D) $2R$

By energy conservation between points A and B

$$Mg(2R) + \frac{1}{2}m(0)^2 = mgH + \frac{1}{2}m(0)^2 \Rightarrow H = 2R$$

Ans.2: (D) 40 sec

$$4t + 2t = 4(60) \Rightarrow t = 40$$

Ans.3: (A) Towards the left

Point of contact of wheel has velocity towards left.

Ans.4: (C) b and m alone

$$F = -\frac{dU}{dx} = -2bx \Rightarrow \omega = \sqrt{\frac{2b}{m}}$$

Ans.5: (D) light is absorbed in quanta of energy $E = h\nu$

Ans.6: (B) 954 kg/m^3

$$V\rho g = \frac{V}{6}(724)g + \frac{5V}{6}(1000)g \Rightarrow \rho = 954 \text{ Kg / m}^3$$

Ans.7: (C) 144 cm

$$n(18) = l \text{ where length of string is } l$$

$$(n+1)(16) = l$$

Gives $n=8$ and $l=144\text{cm}$

Ans.8: (A) $4.8 \times 10^{-4} \text{ C}$

$$Q = \frac{\phi}{R} = \frac{(20 \times 10^{-4})(2.4)}{10} = 4.8 \times 10^{-4} \text{ C}$$

Ans.9: (B) $\frac{3q}{2\sqrt{2}\pi\epsilon_0 a}$

$$V = \frac{kq}{a \sin 45^\circ} + \frac{k(-q)}{a \sin 45^\circ} + \frac{k(3q)}{a \cos 45^\circ} = \frac{1}{4\pi\epsilon_0} \frac{3q}{\frac{a}{\sqrt{2}}} = \frac{3q}{2\sqrt{2}\pi\epsilon_0 a}$$

Ans.10: (D) ∞

$$\text{Resistance} = \frac{dV}{dI} = \frac{1}{\left(\frac{dI}{dV}\right)} = \frac{1}{\text{Slope}} = \frac{1}{0} = \infty$$

Ans.11: (C) It moves back and forth (oscillating) towards the wolf

Sound wave is longitudinal wave .

Ans.12: (B) Silver

Ans.13: (D) $4V$

$$A_1V_1 = A_2V_2 \Rightarrow \pi(2R)^2 V = \pi R^2 V_B \Rightarrow 4V = V_B$$

Ans.14: (B) 6 minutes

$$-\frac{d\theta}{dt} = k[\theta_{av} - \theta_0]$$

$$-\frac{(59-61)}{4} = k\left[\frac{61+59}{2} - 30\right]$$

$$\frac{1}{2} = k30 \Rightarrow k = \frac{1}{60}$$

$$-\frac{(49-51)}{t} = k\left[\frac{51+49}{2} - 30\right]$$

$$\frac{2}{t} = k(20) \Rightarrow t = 6$$

Ans.15:(C)18000C

$$i = \frac{7.5}{9} \therefore Q = it = \frac{7.5}{9}(6)(60)(60) = 18000C$$

Ans.16: (B) $\frac{B\omega l^2}{2}$

Ans.17: (A) $3\Phi_E$

$$\Phi_E = \frac{q}{\epsilon_0} \therefore \Phi = \frac{q + (-3q) + 5q}{\epsilon_0} = \frac{3q}{\epsilon_0} = 3\Phi_E$$

Ans.18:(B)27A

$$IV = P_1 + P_2 + P_3$$

$$I(120) = 1800 + 1300 + 100 \therefore I = 26.67A$$

Ans.19: (A)2A

$$B = \frac{\mu_0}{2\pi(0.1)}[10 + 8 + I - 20] = 0 \therefore I = 2$$

Ans.20:(C)80V

$$|\mathcal{E}| = L \left| \frac{di}{dt} \right| = (40 \times 10^{-6}) \left(\frac{6-0}{3 \times 10^{-6}} \right) = 80V$$

Ans.21: (D) 12.1eV

$$\Delta E = E_3 - E_1 = -1.5 - (-13.6) = 12.1eV$$

Ans.22: (D) There is no change

$$\vec{F} = q\vec{v} \times \vec{B} = 0 \text{ So velocity is constant}$$

Ans.23: (B) -10^{-5}

Ans.24: (A) 1

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m \left(\sqrt{\frac{GM}{r}} \right)^2 = \frac{1}{2}m \frac{GM}{r}$$

$$U = -m \frac{GM}{r} \therefore E = K.E + U = m \frac{GM}{2r} - m \frac{GM}{r} = -m \frac{GM}{2r}$$

Alternative:

$$\text{we know that } E = -K \Rightarrow |E| = K$$

Ans.25:(B) $16 m / s^2, 4 m / s$

$$a_c = 32 \cos 60^\circ = 16 m / s^2$$

$$a_c = \frac{v^2}{R} \Rightarrow 16 = \frac{v^2}{1} \Rightarrow v = 4 m / s$$

Ans.26:(D) $10 \frac{m}{s^2}$ upwards the incline

$$a = \frac{75 - 5g \sin 30^\circ}{5} = (75 - 25) / 5 = 10 m / s^2$$

Ans.27:(A) 60J

$$W = KE_f - KE_i = \frac{1}{2}(3)(64 + 16) - \frac{1}{2}(3)(36 + 4) = \frac{1}{2}(3)(80 - 40) = 60J$$

Ans.28: (C) 335J

$$W = Q_A - Q_R$$

$$25 = 360 - Q_R \therefore Q_R = 335J$$

Ans.29: (A) $\frac{3\sigma}{2\epsilon_0}$

$$E = \frac{\sigma}{2\epsilon_0} + \frac{-2\sigma}{2\epsilon_0} + \frac{4\sigma}{2\epsilon_0} = \frac{3\sigma}{2\epsilon_0}$$

Ans .30: (C) Three in parallel

$$U = \frac{1}{2} CV^2 \quad \text{For } U \text{ maximum, } C \text{ must be maximum}$$

Ans.31:(D) $\frac{20}{3} \Omega$

$$\text{By Wheatstone bridge } R_{eq} = \frac{(4+6)(8+12)}{(4+6)+(8+12)} = \frac{20}{3} \Omega$$

Ans.32: (C) $a < b, b > c$

Ans.33: (B) $2f$

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R} - \frac{1}{-R} \right)$$

$$\frac{1}{f_1} = (\mu - 1) \left(\frac{1}{R} - \frac{1}{\infty} \right) \Rightarrow f_1 = 2f$$

Ans.34: (B) 26V

$$V = (2 + 4)4 + 2 = 26 \text{ volt}$$

Ans.35: (C) $2\sqrt{2}$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mKE}} = \frac{h}{\sqrt{2mqV}}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{\sqrt{m_2 q_2}}{\sqrt{m_1 q_1}} = \frac{\sqrt{4m_p 2e}}{\sqrt{m_p e}} = 2\sqrt{2}$$

Ans.36:(A) $\frac{6g}{5L} \sin \theta$

$$I = 4m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 = \frac{5}{4}mL^2$$

$$\tau = 4mg \frac{L}{2} \sin \theta - mg \frac{L}{2} \sin \theta = mg \frac{3L}{2} \sin \theta$$

$$\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I} = \frac{6g}{5L} \sin \theta$$

Ans.37: (C) $4\hat{i} - 5\hat{j}$

Horizontal component remains constant, whereas vertical component changes its sign.

Ans.38: (C) 5 %

$$T = 2\pi \sqrt{\frac{l}{g}} \Rightarrow \frac{\Delta T}{T} = \frac{\Delta l}{2l} + \frac{\Delta g}{2g} \Rightarrow \frac{\Delta T}{T} \% = \left(\frac{3}{2} + \frac{7}{2}\right) \% = 5\%$$

Ans.39: (D) 100W

$$\text{Work per cycle} = \frac{1}{2} \times (30 - 10)(8 - 2) = 60J \therefore P = \frac{60 \times 100}{60} = 100W$$

Ans.40: (A) Path -l

Ans.41: (A) 3Hz

$$v_1 = 30300 / 100 = 303Hz, v_2 = 30300 / 101 = 300Hz \Rightarrow v_1 - v_2 = 3Hz$$

Ans.42: (C) $0.75I_0$

$$I = I_0 \cos^2 30^\circ = 0.75I_0$$

Ans.43: (B) laser light is highly coherent

Ans.44: (B) 19%

$$KE_2 = \frac{p_2^2}{2m} = \frac{(0.9p)^2}{2m} = \frac{0.81p^2}{2m}$$

Ans.45: (A) Magnification of microscope is inversely proportional to the least distance of distinct vision.

$$\text{Magnification } M = 1 + \frac{D}{f}$$

Ans.46: (C) $64\pi SR^2$

$$W = S \left[8\pi S(3R)^2 - 8\pi S(R)^2 \right] = 64\pi S(R)^2$$

Ans.47: (C) Less than 300 km/hr

$$\langle v \rangle = \frac{d + d}{t_1 + t_2} = \frac{200 + 200}{\frac{200}{400} + \frac{200}{200}} = \frac{800}{3} = 267 \text{ km/hr}$$

Ans.48: (C) remains constant

$$dS = \frac{dQ}{T} = 0 \therefore S = \text{constant}$$

Ans.49: (C) $A = 0, B = 1, C = 1$

Output $C = A + AB$

Ans.50: (C) chromatic aberration

Chemistry

Ans.51: (C) He⁺

$$\text{Ionization Potential} = E_{\infty} - E_1$$

$$54.4 = 0 - E_1 \text{ or } E_1 = -54.4 \text{ eV}$$

$$\text{But } E_1 = -13.6 \times \frac{Z^2}{(1)^2} \text{ eV or } -54.4 \text{ eV} = -13.6 \times Z^2 \text{ or } Z=2 \text{ ,So He}^+ \text{ ion}$$

Ans.52:(C) $n = 3, l = 2, m = 1, s = +\frac{1}{2}$

Energy $\propto (n+l)$

For Options: (A) $(n+l) = 3+0 = 3$

(B) $(n+l) = 3+1 = 4$

(C) $(n+l) = 3+2 = 5$

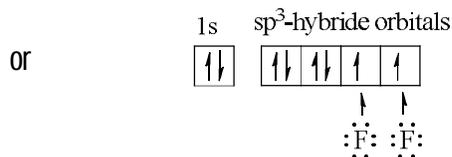
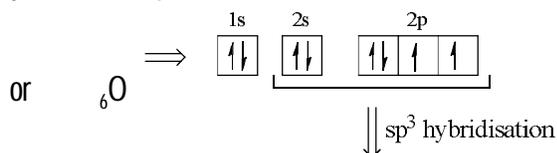
(D) $(n+l) = 4+0 = 4$

So $n = 3, l = 2, m = 1, s = +\frac{1}{2}$ Set of quantum number has highest energy.

Ans.53: (C) sp³

OF₂ :-

$${}_6\text{O} \Rightarrow 1s^2 2s^2 2p^4$$



 sp³, Two lone pairs of electron V-shape

Ans.54.: (D) SO₃²⁻, ClO₃⁻ and BO₃³⁻

NO₃⁻ ⇒ sp² ⇒ Trigonal planar

AsO₃³⁻ ⇒ sp³ ⇒ Pyramidal (one lone pair)

CO₃²⁻ ⇒ sp² ⇒ Trigonal planar

ClO₃⁻ ⇒ sp² ⇒ Pyramidal (one lone pair)

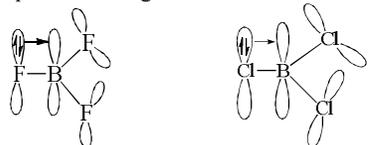
SO₃²⁻ ⇒ sp³ ⇒ Pyramidal (one lone pair)

BO₃³⁻ ⇒ sp³ ⇒ Pyramidal (one lone pair)

So SO₃²⁻, ClO₃⁻ & BO₃³⁻ all are non-planar

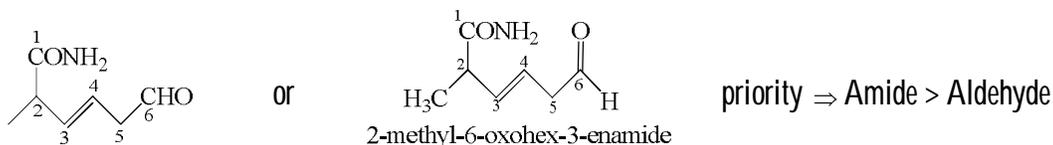
Ans.55: (B) stronger 2p(B)-2p(F) π - bonding

πp-πp back bonding

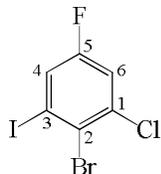


Size of Cl is more than the size of F so in case of BF₃ strong 2p(B)-2p(F) π -bonding occurs so lewis acidity of BF₃ is less than BCl₃.

Ans.56: (A) 2-methyl-6-oxohex-3-enamide



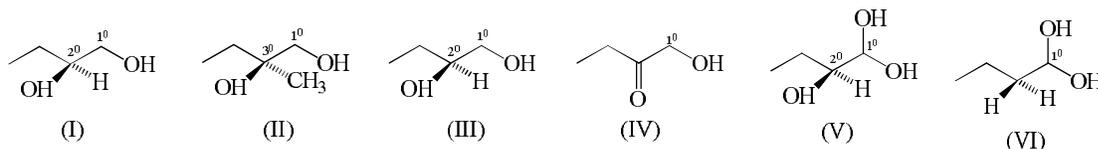
Ans.57: (B) 2-Bromo-1-chloro-5-fluoro-3-iodo benzene



* Numbering according to lowest set of locant rule

2-Bromo-1-chloro-5-fluoro-3-iodo-benzene

Ans.58: (D)(i), (iii), (v)



So at least one 2°- alcohol present in I, III & V

Ans.59: (C) intermediate 2

According to Hammonds Postulates the transition state resemble to that species which is energetically near to it.

Ans.60: (B) Cl > F > Br > I

On moving up to down in the group. Electron affinity decrease due to decrease in size but chlorine has high electron affinity fluorine due to presence of vacant d-orbitals.

Ans.61: (B) Coordination isomerism

Answer is (B) because of coordination isomerism is a form of structural isomerism in which the composition of the complex ion varies. In a coordination isomer the total ratio of ligand to metal remains the same, but the ligands attached to specific metal ion change.

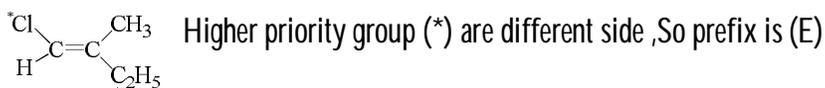
Ans.62: (A) zero

Species which is excess in reaction mixture follow zero order kinetics, so order of reaction with respect to O₂ is zero

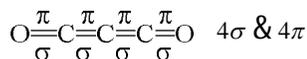
Ans.63: (D) Reduction

Friedel-Craft reaction is a aromatic electrophilic substitution. So reduction is not a fried-craft reaction.

Ans.64: (A) E



Ans.65: (A) 4 σ and 4 π bonds



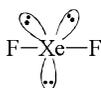
Ans.66: (B) linear, pyramidal

XeF₂ = sp³d hybridization, 3l.p. & 2 l.p.

NH₃ = sp³ hybrid

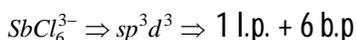
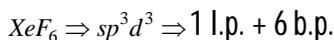
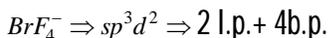
1l.p. + 3b.p.





So

Ans.67: (C) 2,1 and 1



Ans.68: (A) isotropic

Crystalline solids are anisotropic not isotropic

Ans.69: (A) vapour pressure of solute is zero

Non volatile solute is always have zero vapour pressure

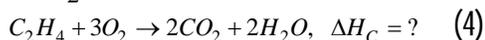
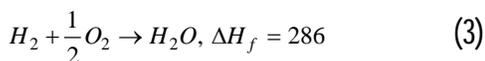
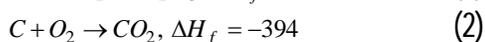
Ans.70: (B) associated colloids

Micelles are associated colloids which are formed above the CMC (critical micelles concentration)

Ans.71: (A) Milk fat is dispersed in water

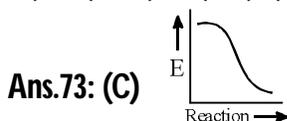
Emulsions are colloids in which both dispersed phase & dispersion medium are liquids. So milk is emulsion in which liquid is dispersed in water.

Ans.72: (D) $-1412 \text{ kJ mol}^{-1}$



But $\text{equ. } 2 \times (\text{equ-2}) - 2 \times (\text{equ-3}) - (\text{equ-1}) = \text{equ-4}$

$$2(-394) + 2(-286) - (52) = -1412 \text{ KJmol}^{-1}$$



If the difference between energy of reactant & transition state is zero then activation energy is zero.

Ans.74: (C) $t_{1/2} \propto a^0$

$$t_{1/2} \propto \frac{1}{n-1}$$

For first order reaction $n = 1$

So $t_{1/2} \propto \frac{1}{a^0}$

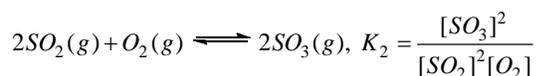
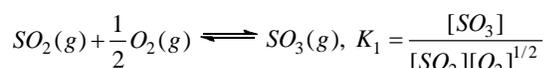
Or $t_{1/2} \propto a^0 \text{ constant}$

Ans.75: (D) 2.0 ML^{-1}

Active mass is concentration in mole litre⁻¹ or concentration in molarity

$$\text{So Molarity} = \frac{8.5}{17} \times \frac{1000}{250} = 2.0 \text{ ML}^{-1}$$

Ans.76: (C) $K_1^2 = K_2$



$$K_1^2 = \frac{[SO_3]^2}{[SO_2]^2[O_2]} = K_2$$

So $K_1^2 = K_2$

Ans.77: (B) three stereoisomers

When same groups are present in opposite side called threo stereoisomer .

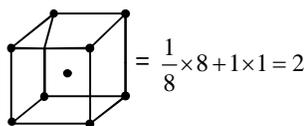
Ans.78: (C) Schottky

During the Schottky defects same number of cations & anions are missing from their lattice site so density is decreased.

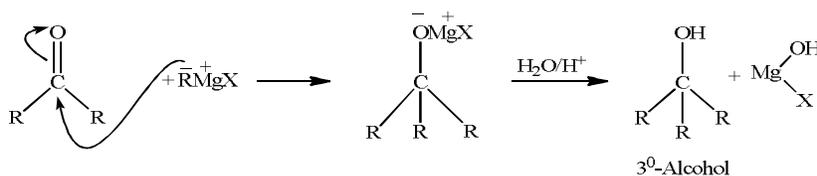
Ans.79: (A) $\frac{1}{8}$

$$N = N_0 / 2^n \therefore N = N_0 / 2^3 = N_0 / 8$$

Ans.80: (B) 2



Ans.81: (C) 3° alcohol



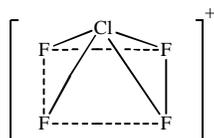
Ans.82: (B) CaOCl₂

Bleaching powder is CaOCl₂

Ans.83: (B) square pyramidal

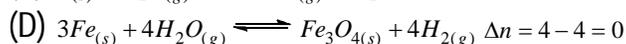
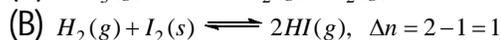
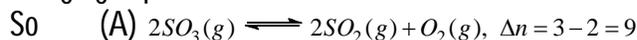


4 b.p. of e⁻ & 1 lone pair of e⁻ & shape is square pyramidal



Ans.84: (D) $3Fe(s) + 4H_2O(g) \rightleftharpoons Fe_3O_4(s) + 4H_2(g)$

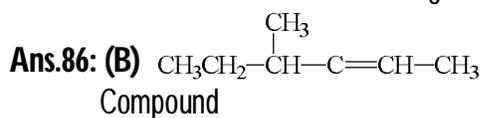
If gaseous moles of reactant is equal to the gaseous moles of product then reaction is not affected by the changing in pressure

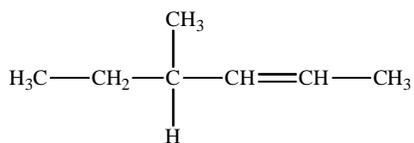


Ans.85: (A) Increasing the temperature

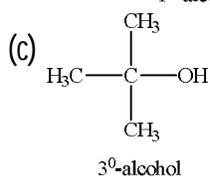
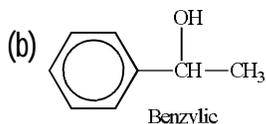
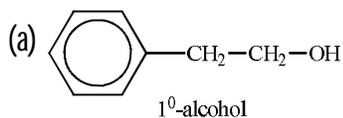


Reaction is exothermic so on increasing the temperature equilibrium shifted in backward direction



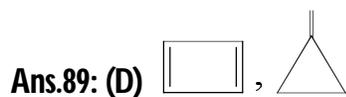


gives geometrical isomerism & it is also give enantiomerism.

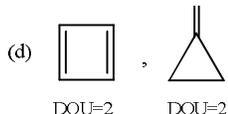
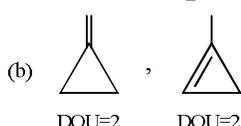
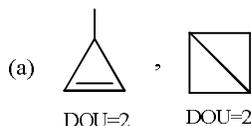


So compound give fastest reaction with conc. HCl

Ans.88: (A) Polythene

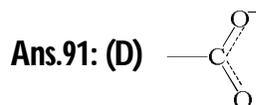
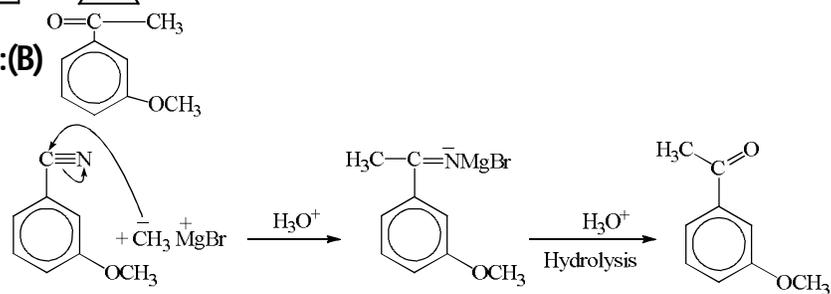


$$\text{C}_4\text{H}_6 \text{ Degree of unsaturation (DOU)} = \frac{10-6}{2} = 2$$

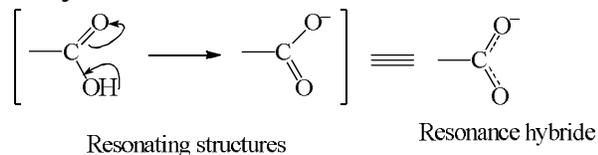


So , is not the pair of C_4H_6

Ans.90: (B)



Resonance in carboxylate ion $-\text{COO}^-$



Ans.92: (B) kg. ms⁻²

$$E = mc^2 \Rightarrow kg(ms^{-1})^2 = Kgm^2s^{-2}$$

So kg.ms⁻² is not the unit of energy .

Ans.93: (A) 134.1 gm mol⁻¹

$$P_{total} = 99.652 KPa$$

$$P_{water} = 85.140 KPa$$

$$P_{liquid} = (99.652 - 85.140) KPa = 14.512 kPa$$

$$\text{And } \frac{m_A}{m_B} = \frac{1.27 g}{1g}$$

$$\text{We have } \frac{m_A}{m_B} = \frac{P_A M_A}{P_B M_B}$$

$$\text{or } M_A = \left(\frac{m_A}{m_B} \right) \left(\frac{P_B M_B}{P_A} \right) \quad \therefore M_A = (1.27) \left(\frac{85.140 KPa \times 18 g mol^{-1}}{14.512 kPa} \right) \cong 134.1 g mol^{-1}$$

Ans.94: (A) Cell will swell

Osmotic pressure

Ans.95: (C) 6.92

Solution is very dilute so concentration of H⁺ ions in HCl solution

= H⁺ ions in water + H⁺ is ion in HCl

$$= 1 \times 10^{-7} + 2 \times 10^{-8} = 12 \times 10^{-8}$$

$$\text{So } pH = -\log(12 \times 10^{-8}) = -\log(2^2 \times 3 \times 10^{-8})$$

$$= -2 \log 2 - \log 3 + 8 \log 10 = -2(0.301) - 0.477 + 8 = 6.92$$

Ans.96: (C) A₃B₁₂C

A	B	C
At corner	At Centre of Each face	At corner
$6 \times \frac{1}{8}$	$6 \times \frac{1}{2}$	$2 \times \frac{1}{8}$
$\frac{3}{4}$	3	$\frac{1}{4}$
3	12	1

So molecular formula = A₃B₁₂C,

Ans.97: (C) X₂Y₄Z

Z	Y	X
Corner	in $\frac{1}{2}$ Td in $\frac{1}{2}$ Oh	
	Voids	voids
$8 \times \frac{1}{8}$	$8 \times \frac{1}{2} \times 1$	$4 \times \frac{1}{2} \times 1$
1	4	2

So formula is X₂Y₄Z

Ans.98: (D) B > A > C

According to question the position of elements in electrochemical series is

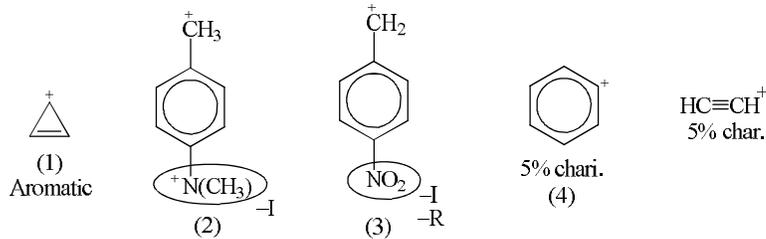
C

A

B

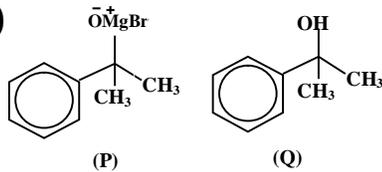
Oxidizing power of elements increases in electrochemical series on moving up to down so decreasing order of oxidizing power is B > A > C

Ans.99: (D) 1 > 2 > 3 > 4 > 5



So decreasing order of stability 1 > 2 > 3 > 4 > 5

Ans.100: (A)



Maths

Ans.101: (D) 90°

Let the angle between P and Q be α . Then as resultant of P and Q is P. $\Rightarrow P^2 = P^2 + Q^2 + 2PQ \cos \alpha$
 $0 = Q + 2P \cos \alpha$

Let θ be the angle which the new resultant makes with Q

Then $\tan \theta = 2P \sin \alpha / (Q + 2P \cos \alpha) = 1/0 = \infty$

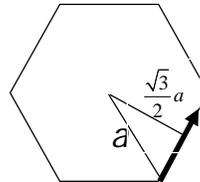
$\Rightarrow \theta = 90^\circ$

Ans.102. (B) $\frac{3L}{4}$

$$x_{cm} = \frac{\int_0^L x dm}{\int_0^L dm} = \frac{\int_0^L x \lambda dx}{\int_0^L \lambda dx} = \frac{\int_0^L x \alpha x^2 dx}{\int_0^L \alpha x^2 dx} = \frac{3L}{4}$$

Ans.103. (B) $\frac{\sqrt{3}}{2} aF$

$$\tau = \left(\frac{\sqrt{3}}{2} a\right) F - \left(\frac{\sqrt{3}}{2} a\right) F + \left(\frac{\sqrt{3}}{2} a\right) F = \left(\frac{\sqrt{3}}{2} a\right) F$$



Ans.104: (A) a

$$a_x = \frac{d^2 x}{dt^2} = \frac{d}{dt}(a(1 + \cos t)) = -a \sin t$$

$$a_y = \frac{d^2 y}{dt^2} = \frac{d}{dt}(a \sin t) = a \cos t$$

$$\text{acceleration} = \sqrt{a_x^2 + a_y^2} = a$$

Ans.105: (B) -2

$$v = \frac{dx}{dt} = \frac{1}{2}t^{-\frac{1}{2}}$$

$$a = \frac{dv}{dt} = \frac{1}{2} \left(-\frac{1}{2} \right) t^{-\frac{3}{2}} = -2 \frac{1}{2^3} t^{-\frac{3}{2}} = -2v^3$$

Ans.106: (A) $\frac{gh}{d}$

$$u = \sqrt{2gh}$$

$$\Rightarrow 0 = u^2 - 2ad$$

$$2gh = 2ad \Rightarrow a = \frac{gh}{d}$$

Ans.107. (C) (4, -8)

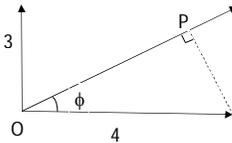
Here $a = 4, m = \tan 45^\circ = 1$

So required normal point $(am^2, -2am) = (4, -8)$

Ans.108: (A) 3.2

$$\tan \phi = 3/4 \Rightarrow \cos \phi = 4/5$$

$$\text{Projection} = 4 \cos \phi = 3.2$$



Ans.109: (D) None of the options

\vec{R} must be perpendicular to \vec{A} as well as perpendicular to $(\vec{B} \times \vec{C})$

Let $\vec{A} = \hat{i}, \vec{B} = \hat{i} + \hat{j}, \vec{C} = \hat{k}$

$$\vec{R} = \vec{A} \times (\vec{B} \times \vec{C}) = \hat{i} \times [(\hat{i} + \hat{j}) \times \hat{k}] = \hat{i} \times [(-\hat{j} + \hat{i})] = -\hat{k}$$

Hence \vec{R} is neither parallel nor perpendicular to \vec{B}

Ans. 110.(C) $e^y = 2e^x + \frac{x^3}{3} + c$

$$e^y \frac{dy}{dx} = 2e^x + x^2$$

$$\int e^y dy = \int (2e^x + x^2) dx$$

$$\Rightarrow e^y = 2e^x + \frac{x^3}{3} + c$$

Ans.111:(A) $y^3 + cy = x$

$$2y^3 dy = ydx - xdy$$

$$2ydy = \frac{ydx - xdy}{y^2} = d\left(\frac{x}{y}\right)$$

$$\Rightarrow y^2 + c = \frac{x}{y}$$

Ans.112: (A) $\frac{1}{3}$

$$\lim_{n \rightarrow \infty} \frac{1}{n^3} (1^2 + 2^2 + 3^2 + \dots + n^2) = \lim_{n \rightarrow \infty} \frac{1}{n^3} \frac{n(n+1)(2n+1)}{6}$$

$$= \lim_{n \rightarrow \infty} \frac{\left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right)}{6} = \frac{2}{6} = \frac{1}{3}$$

Ans.113: (C) 0

$$f(0) = \lim_{x \rightarrow 0} f(x)$$

$$\Rightarrow a = \lim_{x \rightarrow 0} x \sin \frac{1}{x}$$

$$\Rightarrow a = 0 \times (-1 \text{ to } +1) = 0$$

Ans.114: (C) $x^{\sin x} \left(\cos x \log x + \frac{\sin x}{x} \right)$

$$\log y = \log \left(x^{\sin x} \right) = \sin x \log x$$

$$\frac{1}{y} \frac{dy}{dx} = \left(\cos x \log x + \frac{\sin x}{x} \right)$$

$$\Rightarrow \frac{dy}{dx} = x^{\sin x} \left(\cos x \log x + \frac{\sin x}{x} \right)$$

Ans.115: (D) $x - y = 2$ and $x - y = \frac{86}{27}$

Only in (D) option slopes of both lines (tangents) are 1 that is equal to slope of $y = x$ line

Alternative method:

$$\frac{dy}{dx} = 3x^2 - 4x + 1 \quad \dots(1)$$

$$y = x \Rightarrow \frac{dy}{dx} = 1 \quad \dots(2)$$

From (1) and (2) $1 = 3x^2 - 4x + 1 \Rightarrow x = 0, 4/3$

$x = 0$ gives $y = -2$ and $x = 4/3$ gives $y = -50/27$

Thus the tangents to the curve at the points $(0, -2)$ and $(4/3, -50/27)$ are parallel to line $y = x$. The equations of these tangents are $y - (-2) = 1(x - 0)$ and $y - (-50/27) = 1(x - 4/3)$

i.e. $x - y = 2$ and $x - y = \frac{86}{27}$

Ans.116: (A) 1

$$\lim_{x \rightarrow 0} \frac{\cosh x - \cos x}{x \sin x} = \lim_{x \rightarrow 0} \left\{ \frac{\cosh x - \cos x}{x^2} \left(\frac{x}{\sin x} \right) \right\} \quad \left[\frac{0}{0} \text{ form} \right]$$

$$= \lim_{x \rightarrow 0} \left\{ \frac{\cosh x - \cos x}{x^2} \cdot 1 \right\} \quad \left[\frac{0}{0} \text{ form} \right]$$

$$= \lim_{x \rightarrow 0} \left\{ \frac{\sinh x + \sin x}{2x} \right\} \left[\frac{0}{0} \text{form} \right]$$

$$= \lim_{x \rightarrow 0} \left\{ \frac{\cosh x + \cos x}{2} \right\} = \frac{1+1}{2} = 1$$

Alternative:

$$\lim_{x \rightarrow 0} \frac{\cosh x - \cos x}{x \sin x} = \lim_{x \rightarrow 0} \left\{ \frac{\frac{e^x + e^{-x}}{2} - \left(1 - \frac{x^2}{2} + \dots\right)}{x \left(x - \frac{x^3}{3} + \dots\right)} \right\}$$

$$= \lim_{x \rightarrow 0} \left\{ \frac{\frac{\left(1 + \frac{x^2}{2} + \dots\right) + \left(1 - \frac{x^2}{2} + \dots\right)}{2} - \left(1 - \frac{x^2}{2} + \dots\right)}{x \left(x - \frac{x^3}{3} + \dots\right)} \right\} = \lim_{x \rightarrow 0} \left\{ \frac{1}{\left(1 - \frac{x^2}{3} + \dots\right)} \right\} = 1$$

Ans.117: (B) $e^{\left(\frac{1}{e}\right)}$

$$f(x) = (1/x)^x$$

$$f'(x) = (1/x)^x (-\log x - 1)$$

$$f'(x) = 0 \Rightarrow (-\log x - 1) = 0 \Rightarrow x = 1/e$$

$$\text{Maxima of function is} = \left(\frac{1}{1/e}\right)^{1/e} = e^{1/e}$$

$$\text{Here } f''(x) = (1/x)^x (\log x + 1)^2 + (1/x)^x \left(-\frac{1}{x}\right)$$

$$\text{At } x = 1/e, f''(1/e) < 0$$

$$x = 1/e \text{ पर } f''(1/e) < 0$$

Ans.118: (D) $\frac{\pi}{4} - \frac{1}{2} \log 2$

$$\text{Let } x = \sin \theta \Rightarrow dx = \cos \theta d\theta \Rightarrow x = 0, \theta = 0 \text{ and } x = \frac{1}{\sqrt{2}}, \theta = \frac{\pi}{4}$$

$$I = \int_0^{\frac{\pi}{4}} \theta \sec^2 \theta d\theta = [\theta \tan \theta]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} 1 \cdot \tan \theta d\theta$$

$$= \left[\frac{\pi}{4} \tan \frac{\pi}{4} - 0 \cdot \tan 0 \right] - [\log \sec \theta]_0^{\frac{\pi}{4}}$$

$$= \left[\frac{\pi}{4} \right] - \left[\log \sec \frac{\pi}{4} - \log \sec 0 \right] = \frac{\pi}{4} - (\log \sqrt{2} - \log 1) = \frac{\pi}{4} - \frac{1}{2} \log 2$$

Ans.119: (D) $\frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{1}{\sqrt{3}} \tan \frac{x}{2} \right) + c$

$$\int \frac{1}{2 + \cos x} dx = \int \frac{dx}{1 + 2 \cos^2 \frac{x}{2}} = \int \frac{dx}{3 \cos^2 \frac{x}{2} + \sin^2 \frac{x}{2}} = \int \frac{\sec^2 \frac{x}{2}}{3 + \tan^2 \frac{x}{2}} dx$$

Let $\tan \frac{x}{2} = t \Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dt$

$$I = 2 \int \frac{dt}{3 + t^2} = \frac{2}{\sqrt{3}} \tan^{-1} \frac{t}{\sqrt{3}} + c = \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{1}{\sqrt{3}} \tan \frac{x}{2} \right) + c$$

Ans.120.(A) $\frac{\sqrt{7}}{4}$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$e = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$$

Ans.121. (C) $x^2 - y^2 = 32$

Distance between foci = $2ae$

and $16 = 2a\sqrt{2} \Rightarrow a = 4\sqrt{2}$

$$b^2 = a^2(e^2 - 1) = 32(2 - 1), b = 4\sqrt{2}$$

Required equation $\frac{x^2}{32} - \frac{y^2}{32} = 1$

Ans.122: (C) $2x - 3y + 13 = 0$

Equation of chord $T = S'$

$$T = -2x + 3y - 81$$

$$S' = 4 + 9 - 81 = -68$$

Equation of chord $-2x + 3y - 81 = -68$

$$2x - 3y + 13 = 0$$

Alternative

$$xx_1 + yy_1 = x_1^2 + y_1^2$$

$$-2x + 3y = (-2)^2 + (3)^2 = 13$$

Ans.123: (D) $2m^2 = ln$

Eliminating x between the given equations

$$y^2 = 8 \left(-\frac{n + my}{l} \right) \Rightarrow ly^2 + 8my + 8n = 0$$

Given straight line touches the parabola if roots of the equation are same

$$(8m)^2 = 4 \cdot 1 \cdot 8n \Rightarrow 2m^2 = ln$$

Ans.124: (C) $x + 3y = 0$

The centre C of the circle is given by $\left[-\frac{1}{2}(-6), -\frac{1}{2}(2)\right]$ or $(3, -1)$

Required diameter is the line joining the origin $(0,0)$ and the centre $C(3,-1)$ and hence the required equation is

$$y - 0 = \frac{-1 - 0}{3 - 0}(x - 0)$$

$$3y = -x \Rightarrow x + 3y = 0$$

Ans.125: (B) $|z + 5|^2$

$$(z + 5)(\bar{z} + 5) = (z + 5)\overline{(z + 5)} = |z + 5|^2$$

Ans.126: (D) $(z \bar{z})$ is nonnegative real

Ans.127: (B) -3

$$\therefore 1 + \omega + \omega^2 = 0$$

$$\therefore (1 + \omega - \omega^2)^2 + (1 - \omega + \omega^2)^2 + 1 = (-\omega^2 - \omega^2)^2 + (-\omega - \omega)^2 + 1$$

$$= 4\omega^4 + 4\omega^2 + 1 = 4(\omega + \omega^2) + 1 = 4(-1) + 1 = -4 + 1 = -3$$

Ans.128: (A) 0

$$(1 + i\sqrt{3})^{12} = 2^{12} \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)^{12} = 2^{12} (\cos 4\pi + i \sin 4\pi) = 2^{12} + i \cdot 0$$

Ans.129: (D) $f(\theta) \geq 4$

$$f(\theta) = 2 \left[(\cos \theta - \sec \theta)^2 + 2 \cos \theta \sec \theta \right] = \left[2(\cos \theta - \sec \theta)^2 + 4 \right] \geq 4$$

Ans.130: (C) $x = \frac{n\pi}{2} + \frac{\pi}{8}$

$$\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = 2 \Rightarrow \cos^2 x - \sin^2 x = 2 \sin x \cos x$$

$$\Rightarrow \cos 2x = \sin 2x$$

$$\Rightarrow \tan 2x = 1 \Rightarrow 2x = n\pi + \frac{\pi}{4}$$

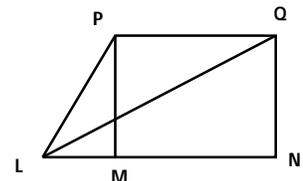
Ans.131: (A) $\frac{100}{\sqrt{3}} m/s$

$$PQ = MN = LN - LM = 1 \cot 30^\circ - 1 \cot 60^\circ = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}} km$$

$$Speed = \frac{2000}{\sqrt{3}} \cdot \frac{1}{20} = \frac{100}{\sqrt{3}} m/s$$

Ans.132: (D) $\frac{1}{4} \sin \theta \sin 4\theta$

$$\sin^2 \theta \cos \theta (\cos^2 \theta - \sin^2 \theta) = \sin^2 \theta \cos \theta \cos 2\theta$$



$$= \frac{1}{2} \sin \theta \sin 2\theta \cos 2\theta = \frac{1}{4} \sin \theta \sin 4\theta$$

Ans.133: (A) Isosceles triangle

$$2 \sin C \cos A = \sin B \Rightarrow 2 \cos A = \frac{\sin B}{\sin C}$$

$$2 \frac{b^2 + c^2 - a^2}{2bc} = \frac{b}{c} \Rightarrow b^2 + c^2 - a^2 = b^2 \Rightarrow c^2 = a^2$$

Ans.134:(C) $\frac{1}{\sqrt{5}}$

$$\tan \left[\frac{1}{2} \cos^{-1} \left(\frac{2}{3} \right) \right] = \tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta}$$

$$= \frac{1 - \cos \theta}{\sqrt{1 - \cos^2 \theta}} = \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \sqrt{\frac{1 - \frac{2}{3}}{1 + \frac{2}{3}}} = \sqrt{\frac{1}{5}}$$

Ans.135: (D) $\phi = \frac{xy}{zr}$

$$\tan^{-1} \frac{yz}{xr} + \tan^{-1} \frac{xz}{yr} = \tan^{-1} \left[\frac{\frac{yz}{xr} + \frac{xz}{yr}}{1 - \frac{z^2}{r^2}} \right] = \tan^{-1} \left[\frac{\frac{z}{xyr} (x^2 + y^2)}{1 - \frac{z^2}{r^2}} \right]$$

$$= \tan^{-1} \left[\frac{\frac{zr^2}{xyr} (x^2 + y^2)}{x^2 + y^2 + z^2 - z^2} \right] = \tan^{-1} \left[\frac{zr}{xy} \right] = \cot^{-1} \left[\frac{xy}{zr} \right] = \frac{\pi}{2} - \tan^{-1} \left[\frac{xy}{zr} \right]$$

Ans.136: (B) $\frac{2}{7}$

$$\text{Total numbers } {}^7P_5 = 2520$$

$$\text{Total ways of odd digits at both ends} = {}^4P_2 = 12$$

$$\text{Total ways of writing digits at remaining 3 places} = {}^5P_3 = 60$$

$$\text{Total favourable conditions} = 12 \times 60 = 720$$

$$\text{Required probability} = \frac{720}{2520} = \frac{2}{7}$$

Ans.137: (D) $\left(\frac{9}{10} \right)^5$

$$\text{Probability of cycle having no puncture is } \left(\frac{90}{100} \right)$$

By binomial distribution $p = \left(\frac{9}{10}\right), q = \frac{1}{10}$

Required probability = ${}^5C_5 q^0 p^5 = \left(\frac{9}{10}\right)^5$

Ans.138: (C) 3/5

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

Since $P(E_1)$ and $P(E_2)$ are independent

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1)P(E_2)$$

$$= \frac{1}{3} + \frac{2}{5} - \frac{1}{3} \cdot \frac{2}{5} = \frac{9}{15} = \frac{3}{5}$$

Ans.139: (A) $\frac{1}{35}$

Exhaustive events = 7

Alternative manner

MWMWMWM

Total ways = 4 × 3

Required probability = $\frac{4 \times 3}{7} = \frac{1}{35}$

Ans.140. (C) H.P.

$$\log_2 6 = \log_2 (3) + \log_2 2 = \log_2 3 + 1$$

$$\log_2 12 = \log_2 (3) + \log_2 4 = \log_2 3 + 2$$

Hence $\log_2 3, \log_2 6, \log_2 12$ are in A.P.

or $\frac{1}{\log_2 3}, \frac{1}{\log_2 6}, \frac{1}{\log_2 12}$ are in H.P.

Hence $\log_3 2, \log_6 2, \log_{12} 2$ are in H.P.

Ans.141.(C) $2(s-r)$

$$r + t = 2s \Rightarrow t = 2s - r$$

$$t - r = 2(s - r)$$

Ans.142: (B) 1

Ans.143: (D) $p > 3$

$$p = (\log_\pi 3) + (\log_\pi 4) + 1 = \{(\log_\pi 12) + 1\} > (2 + 1)$$

$$\log_\pi 12 > \log_\pi \pi^2 \Rightarrow \log_\pi 12 > 2 \quad \because \log_\pi \pi^2 = 2$$

Ans.144 (A) 252

Total terms = 11

$$\text{Midterm} = T_6 = {}^{10}C_5 \left(\frac{3x^2}{5}\right)^5 \left(\frac{5}{3x^2}\right)^5 = \frac{10!}{5!5!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 252$$

Ans.145: (C) 2

From first eq. $a + b = -1, ab = 1$

From second equation $\frac{a}{b} + \frac{b}{a} = -p, \frac{a}{b} \cdot \frac{b}{a} = q = 1$

$$-p = \frac{a^2 + b^2}{ab} = \frac{(a+b)^2 - 2ab}{1} = (-1)^2 - 2(1) = -1 \Rightarrow p = 1$$

$$p + q = 2$$

Ans.146: (A) 0

$$\Delta = \frac{1}{abc} \begin{vmatrix} 1 & abc & a^4 \\ 1 & bca & b^4 \\ 1 & cab & c^4 \end{vmatrix} = \begin{vmatrix} 1 & 1 & a^4 \\ 1 & 1 & b^4 \\ 1 & 1 & c^4 \end{vmatrix} = 0 \because C_1 = C_2$$

Ans.147: (D) None of the options

$$\begin{bmatrix} 9x-1 \\ 6 \end{bmatrix} + \begin{bmatrix} -2x \\ 3 \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 7x-1 \\ 9 \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \end{bmatrix} \Rightarrow 7x-1=8 \Rightarrow x = \frac{9}{7}$$

Ans.148: (D) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is not unit matrix.

Ans.149: (B) One-one Into function

For any $x_1, x_2 \in N$

$$x_1 \neq x_2 \Rightarrow 2x_1 + 3 \neq 2x_2 + 3$$

$$f(x_1) \neq f(x_2)$$

So function is one one function

$$y = 2x + 3 \Rightarrow x = \frac{y-3}{2}$$

$$f^{-1}(x) = \frac{x-3}{2} \notin N \text{ (Domain) when } x=1,2,3,\dots$$

So function is one one into function.

Ans.150: (B) $[-2, \infty)$

$$\text{Let } f(x) = y \Rightarrow x^2 - 6x + 7 - y = 0$$

$$\because x \text{ is real so } B^2 - 4AC \geq 0$$

$$\Rightarrow 36 - 4(7 - y) \geq 0 \Rightarrow 2 + y \geq 0$$

$$\Rightarrow y \geq -2 \therefore \text{Range} = [-2, \infty)$$